Temporal trend and frequency of maximum precipitations in Cruzeiro do Sul, Acre, Brazil

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ABSTRACT: The verification of temporal trends in historical series and the frequency analysis of maximum precipitations events are important for the understanding of the dynamics of the hydro-meteorological processes. In this context, this study aimed at verifying series trends in the observed data from the annual maximum monthly precipitations in Cruzeiro do Sul, Acre State, Brazil, registered at 82704 station (7°38’S; 72°40’W) operated by the National Meteorological Institute, besides performing the frequency analysis through Gumbel probabilistic distribution for the maximum events. The results obtained through Mann-Kendall test suggest a light increasing temporal trend; however, it converges on rejecting the hypothesis of a significant trend, at the significance level of 5%, considering the analysis between 1971 and 2013. Regarding the frequency analysis, it was shown that Gumbel’s distribution was appropriate to represent the maximum precipitation, through the adherence obtained by the Kolmogorov-Smirnov and the Chi-Square tests, to 5% probability.

Keywords: maximum precipitation series, trend verification, probability distribution.

Tendência temporal e frequência de precipitações máximas em Cruzeiro do Sul, Acre

RESUMO: A verificação de tendências temporais em séries históricas e a análise de frequência da ocorrência de eventos de precipitações máximas são importantes para o conhecimento da dinâmica dos processos hidrometeorológicos. Neste contexto, o presente trabalho objetivou verificar tendências em séries de dados observados de precipitações totais mensais máximas anuais em Cruzeiro do Sul, no estado do Acre, registrados na estação 82704 (7°38’S; 72°40’W), operada pelo Instituto Nacional de Meteorologia, além de realizar a análise de frequência, por meio da distribuição de probabilidades de Gumbel para eventos máximos. Os resultados obtidos por meio do teste de Mann-Kendall sugerem leve tendência temporal ascendente, porém, convergem para a não aceitação da hipótese de tendência significativa, ao nível de significância 5%, considerando-se análise compreendida entre os anos de 1971 e 2013. Quanto à análise de frequência, evidenciou-se que a distribuição de Gumbel foi apropriada para representar as precipitações máximas, por meio da aderência obtida pelos testes de Kolmogorov-Smirnov e do Qui-Quadrado, a 5% de probabilidade.

Palavras-chave: séries de precipitações máximas, verificação de tendências, distribuição de probabilidades.

1. INTRODUCTION

The dynamics involved in hydrological processes always aroused the interest of humanity, that exist evidence to accept the hypothesis that its history is closely linked to the use of resources originating from such processes. Water, designed to meet the basic needs of human beings or other purposes such as industry, agriculture or even be used as a driving force, among others, became a determining factor in the evolution of societies and currently, strongly influences social and economic development of nations (CASTRO, 2007; KOBIYAMA et al., 2008; BAPTISTA, COELHO, 2010). The watercourses and precipitations flow rates are among the main hydrologic variables, which allow a better understanding of the dynamics of hydro-climatic processes acting on the planet, as well as to support a decision for the management of available water resources. According to Chierice Júnior; Landim (2014), the fundamental component of the hydrological cycle is the hydrographic basin defined by an area of natural precipitation catchment, which converges to a single exit point, the outfall. The scientific and technological advances, over the years, have enabled the development of tools able to quantify the frequency and magnitude of occurrence of the important hydro-meteorological phenomenon, such as precipitation. In this
sense, given the randomness involved in the occurrence of these phenomenon’s, statistical methods are especially important.

The verification of trends in temporal series of hydrological observations subsidizes discussions about possible climate changes on different scales, which possibly interfere with rainfalls in both regional and global scale, and enhance the risk associated with the occurrence of extreme events.

Further to this context, most statistical methods developed until now, consider the stationary as one of its baseline assumptions, i.e., not occurring changes in the statistical characteristics over time, the temporally variable records as precipitation. However, there are enough evidence to reject this hypothesis (MILLY et al., 2008; VILLARINI et al., 2010; SILVA et al., 2012; TIBÚRCIO; CORREIA, 2012; SALAS; OBEYSEKERA, 2014). Thus, the prior verification of the presence of possible temporal trends or other non-stationary became fundamental when it proposes to carry out studies of the dynamics of occurrence of these phenomena.

Therefore, the frequency analysis may be considered one of the most used stochastic methods in the field of hydrological sciences and is intended to extract population inferences about the probability associated with a certain variable, from a sample set of its occurrences (KATZ et al., 2002).

Many studies destined for both the verification of trends in series of rainfall observations, as to modeling the frequency of its occurrence are available in the literature, both in global and regional scale. However, there are still gaps on the temporal behavior of these variables, especially in the State of Acre, inserted in the Brazilian Amazon, whose climate setting and distinctive natural diversity, require a better understanding of bioclimatic processes that act on it, which are of great importance to the science. The region of Cruzeiro do Sul, part of the evidence state, shows similar conditions (SOUZA et al., 2011; SENA et al., 2012; SANTOS et al., 2012).

Given the importance of knowledge about the regime of precipitations in Cruzeiro do Sul region, located in the southwest of the Brazilian Amazon, this study aims to verify the trends in series of data observed annual maximum monthly total precipitation in addition to a frequency analysis, by fitting the data to the distribution of Gumbel probabilities for maximum precipitation. However, there are enough evidence to reject the hypothesis (MILLY et al., 2008; VILLARINI et al., 2010; MILLY et al., 2008; VILLARINI et al., 2010; SENA et al., 2012; TIBÚRCIO; CORREIA, 2012; SALAS; OBEYSEKERA, 2014). Thus, the prior verification of the presence of possible temporal trends or other non-stationary becomes fundamental when it proposes to carry out studies of the dynamics of occurrence of these phenomena.

2. MATERIAL AND METHODS

2.1. Study area

The precipitation data from meteorological station 82704 were used in this work, located in the city of Cruzeiro do Sul, Acre, operated by the National Meteorology Institute-INMET (7°38’S, 72°40’W in WGS84 datum), and the total precipitation data recorded for the period between the 1971 and 2013.

The city of Cruzeiro do Sul is located in the Juruá Valley, in the State of Acre. The weather in the region is humid tropical (Af), with an average annual temperature of 24.5 °C (DELGADO; SOUZA, 2014). The region has a pattern with a dry season (May to September), and a particularly rainy season (October to April), with average annual precipitations of about 2,000 mm (SOUZA et al., 2011; MACEDO et al., 2013.).

Figure 1 shows the location of the city of Cruzeiro do Sul, and the cited weather station, whose data are used in this study.

Figure 1. Location of the city of Cruzeiro do Sul, Acre State, and its weather station.

2.2. Mann-Kendall Test

The nonparametric Mann-Kendall test (also called MK Test) is widely used to identify trends in series of hydro-meteorological observations (PORTELA et al., 2011). According to Delgado; Souza (2014) and Wagesho et al. (2012), given a number (X1, X2,..., Xn) from a sample of random, independent and identically distributed variables (iid), the MK test is given by the following equation:

\[ S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sin(\alpha_{ij}) \]

where: \( \alpha_{ij} \) represents the series values (in this case, taken in annual intervals); i and j are the time indices, and n is the number of elements of the series. The signal term \( \alpha_{ij} \) is determined by the following equation:

\[ \sin(\alpha_{ij}) = \begin{cases} +1 & \text{if } X_j > X_i \\ 0 & \text{if } X_j = X_i \\ -1 & \text{if } X_j < X_i \end{cases} \]

Mann (1945) and Kendall (1975) explain that the S statistic follows approximately a normal distribution for \( n \geq 8 \). For data without linked elements (equal values), the test statistic has an average \( E(S) \) and variance \( Var(S) \) given respectively by:

\[ E(S) = 0 \]  \hspace{1cm} (3)

\[ Var(S) = \frac{1}{18} n(n - 1)(2n + 5) \]  \hspace{1cm} (4)

Having linked points, the variance is corrected by the following:

\[ Var(S) = \frac{1}{18} n(n - 1)(2n + 5) - \frac{1}{18} \sum_{p=1}^{n} t_p (t_p - 1)(2t_p + 5) \]  \hspace{1cm} (5)

\( t_p \) is determined by the elements of the series, respectively by:

\[ t_j = \frac{1}{2} \left( \frac{X_j + X_{j+1}}{2} \right) \]  \hspace{1cm} (6)

\[ s_{ij} = \sin(\alpha_{ij}) \]  \hspace{1cm} (7)

where: \( t_j \) is the average of the elements \( X_j \) and \( X_{j+1} \); and \( s_{ij} \) is the signal term.
where \( t_\alpha \) denote the number of indices linked points \( p \) to \( q \).

The significance of the MK test can be verified by a bilateral test with standardized statistical \( Z_{MK} \) expressed as follows:

\[
Z_{MK} = \begin{cases} 
  \frac{S - 1}{\sqrt{\text{Var}(S)}} & \text{se } S > 0 \\
  0 & \text{se } S = 0 \\
  \frac{S + 1}{\sqrt{\text{Var}(S)}} & \text{se } S < 0
\end{cases}
\]  

(6)

The null hypothesis \( (H_0) \), for lack of trend in the series, is rejected if \( |Z_{MK}| > Z_{\alpha/2} \), where \( \alpha \) is the significance level and \( Z_{\alpha/2} \) is the value of the standard distribution normal with an exceedance probability of \( \alpha/2 \). The \( Z_{MK} \) signal indicates whether the trend is growing \( (Z_{MK} > 0) \) or descending \( (Z_{MK} < 0) \). The commonly used significance level is \( \alpha = 0.05 \), as is the case in the present study.

The \( p \)-value of \( S \) statistics for the sample data can be estimated by the cumulative probability of the normal distribution, if \( p \leq \alpha \), the test provides evidence for rejection \( H_0 \), suggesting the presence of trends in the series.

### 2.3. Gumbel Distribution

The Gumbel distribution for maximum or double exponential is one of the probability distributions most commonly used in frequency analysis of hydro-meteorological variables, with numerous applications, especially in studies about the system of maximum precipitations (COLES, 2001; KATZ, 2010). To Back (2001) the acceptance of the Gumbel distribution for frequency analysis of maximum rainfalls is widespread.

Considering a simple random sample \( \{X_1, X_2, \ldots, X_n\} \), which elements are independent and identically distributed (iid), the Cumulative Probability Function (CPF) of the Gumbel distribution is expressed as follows:

\[
F_X(x) = \exp \left[ -\exp \left( -\frac{x - \mu}{\sigma} \right) \right]
\]  

(7)

for \( -\infty < x < \infty \), \( -\infty < \mu < \infty \), \( \sigma > 0 \), where \( \sigma \) and \( \mu \) are the parameters of scale and position, respectively, and \( x \) the value of the independent variable. The density distribution function \( \text{pdf} \) Gumbel is expressed by equation (8), expressed as follows:

\[
f_X(x) = \frac{1}{\sigma} \exp \left[ -\frac{x - \mu}{\sigma} \right] \exp \left( -\frac{x - \mu}{\sigma} \right)
\]  

(8)

In practice, under the protection of the increasing monotonicity property of the logarithmic transformation, it is common to employ the log likelihood function for the parametric estimation effect expressed by the following equation:

\[
\ell(\theta) = \log L(\theta) = \sum_{i=1}^{k} \log f(x_i; \theta_k)
\]  

(14)

### 2.4. Adherence Tests

The adherence tests relate to verification of the form of a probability distribution and indicate the suitability of setting a probability distribution for a set of observed data (NAGHETTINI; PINTO, 2007; SILVINO et al., 2007). Currently, the Chi-Square \( (\chi^2) \) and the Kolmogorov-Smirnov (KS) test are highlighted.

As described by Silvino et al. (2007), \( \chi^2 \) test comparisons between the empirical and theoretical frequencies is carried out using the following equation:

\[
\chi^2 = \sum_{i=1}^{k} \left( \frac{F_{0i} - F_{ei}}{F_{ei}} \right)^2
\]  

(15)
where: \( k \) is the number of classes and \( F_{ei} \) and \( F_{oi} \) is, respectively, the observed and expected frequencies. The rejection decision or not about the probabilistic behavior of the distribution depends on the critical value of the significance level and the degree of freedom of it.

But the Kolmogorov-Smirnov test is based on the maximum difference between the functions of empirical and theoretical cumulative probabilities expressed by the following equation:

\[
D_{obs} = \text{Max} \left| F(x) - G(x) \right|
\]  \hspace{1cm} (16)

where: \( F(x) \) and \( G(x) \) represent, respectively, the theoretical and observed distribution function. If \( D_{obs} < D_{tab} \) should not reject the hypothesis that the theoretical probability distribution may represent the observed data, i.e., the sample data is coming from a population whose shape distribution can be described by the probability distribution being tested.

3. RESULTS AND DISCUSSION

Preliminarily, there was an analysis of data from annual maximum monthly total precipitations, which series consists of 43 years of records, from 1971 to 2013, for the region of Cruzeiro do Sul in Acre State. At this stage, it was verified that the highest value recorded for the period under analysis was 480.2 mm in 1990; while the minimum recorded value was 186.8 mm in 1979. It was also perceived, the variation in the recorded values for the period under study, which 48.84% are above the historical average, which value is 350.1 mm, while 51.16% are under this value (Figure 2).

From the application of Mann-Kendall Test, the value of \( Z_{MK} = 1.80 \), was obtained, indicating increasing temporal trend for the series of annual maximum monthly total precipitation. Figure 3 allows ratification of this result by the linear regression behavior of this variable in time. It stands out (Figure 3) that the dotted line indicates the trend of the time series values. However, the increasing tendency is not significant at the significance level \( \alpha = 0.05 \), since \( Z_{MK} = 1.96 \) and, therefore, \( |Z_{MK}| < Z_{\alpha/2} \). Considering the significance level was set at \( \alpha = 0.05 \), the finding trend is not significant, is reinforced by the p-value obtained in the MK Test (p-value=0.075), which in turn is higher than \( \alpha \) value, indicating thereby the non-acceptance of the significant trend hypothesis in the study series.

Concerning the total monthly precipitation, Delgado et al. (2012), in a study about trends for some climate variables in the Western Amazon, reached similar conclusions about the total monthly precipitation. Blain; Moraes (2011) in a study about daily maximum precipitation series, in the state of São Paulo, except for the Pindorama region, which data consider records ranging from the years 1948 and 2007 also had similar results. Sansigolo (2008) concluded by non-acceptance of significant trend hypothesis in the observed records of precipitation in the region of Piracicaba, State of São Paulo, and warn that possible global or regional climate change cannot have significantly affected the rainfall in that region. Also, similar results were obtained by Delgado; Souza (2014) in Cruzeiro do Sul region.

In the following step, it was sought to perform frequency analysis on the data of annual maximum monthly total precipitation, recorded in the region of Cruzeiro do Sul, through adjusting the Gumbel distribution. Using the maximum likelihood method, the estimated values for the position parameters (\( m = 314.4 \)) and scale (\( s = 70.47 \)) of the Gumbel distribution were obtained.

There was also that the sample data in the study meets the conditions of randomness, independence, homogeneity, and stationary.

Figure 4 presents the settings of the observed values and the empirical quantiles, plotted on a Gumbel probability paper, using the Weibull plot position, with a 95% confidence interval. It was also found, through the Kolmogorov-Smirnov (KS) and \( \chi^2 \) tests, the adherence of the setting data to the Gumbel
distribution, which the results showed p-value of 0.6814 and 0.2033, respectively, converging not rejecting the hypothesis that the observed data were obtained from a Gumbel population, at a significance level of 5% and thus, confirming a good adjustment of the data to the concerned probability distribution.

Blain; Moraes (2011) ensure that knowledge of the probabilities of occurrences of precipitations recorded values, especially those associated with extreme events, is an important step to reduce human’s vulnerability to impacts related to these phenomena. In addition, Back (2001), reports that there is a sufficient theory able to justify the use of a given probability distribution for statistical modeling of hydro-meteorological variables, such as precipitation, however, among other reasons, the author clarifies, that Gumbel distribution has often been reported as the most appropriate to the analysis of extreme values (especially the maximum values), and be such distribution, widely used in work to analyze the temporal behavior of heavy rains.

Sansigolo (2008) obtained a good adjustment to the Gumbel distribution to the observed data of annual maximum daily precipitations for the region of Piracicaba, São Paulo State, which adherence was confirmed by the tests used in this study. Similarly, in a focused study to analyze the temporal frequency of maximum precipitation for the city of Cascavel, Paraná State, Quadros et al. (2011) concluded that the Gumbel distribution was satisfactorily adjusted to the data observed in the region.

In this sense, the equations presented in (9) and (10) are important tools for adjusted quantile data estimation to a Gumbel distribution, regarding the likelihood of overcoming it and associated with its return time. For example, the quantile associated with a return time of T=100 years to the data of the annual maximum monthly total precipitation in Cruzeiro do Sul in the region of Acre State, obtained by the equation expressed in (10) is, approximately 638.5 mm, which value is inserted in a confidence interval [553.4; 723.7], to 95% confidence level (Table 1).

<table>
<thead>
<tr>
<th>Return time (years)</th>
<th>Precipitation (mm)</th>
<th>Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>340.2</td>
<td>[315.5; 364.9]</td>
</tr>
<tr>
<td>5</td>
<td>420.1</td>
<td>[382.1; 458.0]</td>
</tr>
<tr>
<td>10</td>
<td>473.0</td>
<td>[424.3; 521.6]</td>
</tr>
<tr>
<td>25</td>
<td>539.8</td>
<td>[476.7; 602.9]</td>
</tr>
<tr>
<td>69</td>
<td>612.2</td>
<td>[533.0; 691.4]</td>
</tr>
<tr>
<td>100</td>
<td>638.5</td>
<td>[553.4; 723.7]</td>
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<tr>
<td>200</td>
<td>687.6</td>
<td>[591.3; 783.8]</td>
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<tr>
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<td>752.2</td>
<td>[641.2; 863.2]</td>
</tr>
<tr>
<td>1,000</td>
<td>801.1</td>
<td>[678.9; 923.3]</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

The annual maximum monthly total precipitations, recorded in Cruzeiro do Sul, Acre State, have increasing temporal trend, but not significant at the 5% level. It is suggested not to accept the hypothesis that the rainfall of Cruzeiro do Sul region has been significantly affected by potential climate change.

The Gumbel probabilities distribution, to maximum, is appropriate to adjust the precipitation data for Cruzeiro do Sul region, which result is particularly important for planning future actions and provide management elements of the available natural resources.

Furthermore, it is suggested to carry out additional studies that may corroborate the obtained results or make additional findings of rainfall patterns of the locality under study.

5. ACKNOWLEDGMENTS

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6. REFERENCES


